Aufgabe 1 Finde die Lösungen der folgenden Potenzgleichungen.

a) $(x-2)(x+2) = -3$

$$\underbrace{(x-2)(x+2)}_{} = -3$$

$$x^2 - 4 = -3 \quad | +4$$

$$x^2 = 1 \quad | \sqrt{}$$

$$x = \pm 1 \longrightarrow \underline{\mathbb{L} = \{-1, +1\}}$$

b) $4(m+1)^2 = 1 \quad \big| :4$

$$(m+1)^2 = \frac{1}{4} \quad \big| \sqrt{}$$

$$m+1 = \pm\frac{1}{2} \quad \big| -1$$

$$m = -1 \pm \frac{1}{2} \begin{cases} -\frac{1}{2} \\ -\frac{3}{2} \end{cases} \rightarrow \underline{\mathbb{L} = \left\{-\frac{3}{2}, -\frac{1}{2}\right\}}$$

c) $(-x)^3 = -\frac{216}{1000} \quad \Big| \sqrt[3]{}$

$$-x = \sqrt[3]{-\frac{216}{1000}} = \sqrt[3]{-\frac{54}{250}}$$

$$= \sqrt[3]{-\frac{27}{125}} = \sqrt[3]{-\frac{3^3}{5^3}}$$

$$= -\sqrt[3]{\frac{3^3}{5^3}} = -\frac{3}{5}$$

$$\sqrt[3]{-\frac{3^3}{5^3}} = \sqrt[3]{(-1)\cdot\frac{3^3}{5^3}} = \underbrace{\sqrt[3]{-1}}_{-1}\cdot\sqrt[3]{\frac{3^3}{5^3}} = -\frac{3}{5}$$

$$-x = -\frac{3}{5} \quad \Big| \cdot(-1)$$

$$x = \frac{3}{5}$$

$$\rightarrow \underline{\mathbb{L} = \left\{\frac{3}{5}\right\}}$$

d) $a^4 + \frac{3}{a} = -\frac{2\cancel{a}}{a^{\cancel{2}}} \quad \Big| \cdot a$

$$a^5 + 3 = -2 \quad \Big| -3$$

$$a^5 = -5 \quad \Big| \sqrt[5]{}$$

$$a = \sqrt[5]{-5} = -\sqrt[5]{5} \longrightarrow \underline{\mathbb{L} = \{-\sqrt[5]{5}\}}$$

$$\uparrow$$

$$\sqrt[5]{(-1) \cdot 5}$$

e) $x + x(x+3) \;=\; 4(x+3) - 12$

$$x + x^2 + 3x = 4x + \cancel{12} - \cancel{12}$$

$$\cancel{4x} + x^2 = \cancel{4x} \quad \Big| -4x$$

$$x^2 = 0 \quad \Big| \sqrt{}$$

$$x = 0$$

$$\underline{\mathbb{L} = \{0\}}$$

Aufgabe 2 Finde die Lösungen der folgenden Wurzelgleichungen. Achte auf mögliche Scheinlösungen!

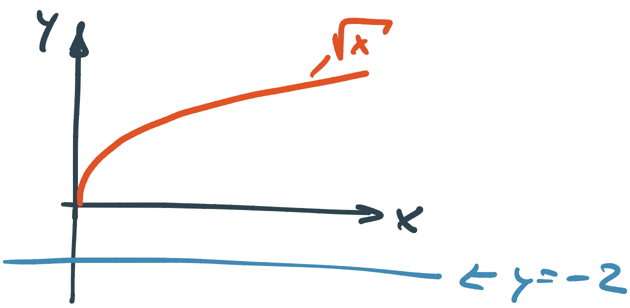
a) $\sqrt{x} = -2 \quad \Big| \ \square^2$

$\cancel{x = 4}$ Scheinlösung!

oben eingesetzt: $\underbrace{\sqrt{4}}_{2} \stackrel{?}{=} -2$

$2 \neq -2$

$\underline{\mathbb{L} = \{\ \}}$ (keine Lösung)



b) $\sqrt{b+1} = \frac{b}{2} + 1 \qquad | \ \square^2$

$b+1 = \left(\frac{b}{2}\right)^2 + 2 \cdot \frac{b}{2} \cdot 1 + 1^2$

$\cancel{b} + \cancel{1} = \frac{b^2}{4} + \cancel{b} + \cancel{1} \qquad | -b \ -1$

$\frac{b^2}{4} = 0 \qquad | \cdot 4$

$b^2 = 0 \quad \rightarrow \quad b = 0$

$\mathbb{L} = \{ 0 \}$

c) $x = \sqrt{3-2x}$ $\quad | \ \square^2$

$x^2 = 3 - 2x \quad | +2x \ -3$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1$ ✓ oder $\cancel{x = -3}$

Scheinlösung!

$\underline{\mathbb{L} = \{1\}}$

d) $\sqrt[3]{\frac{x+1}{5}} = 9^{1/2} = 3 \quad | \square^3$

$\frac{x+1}{5} = 3^3 = 27 \quad | \cdot 5$

$x+1 = 135 \quad | -1$

$x = 134$ ✓

$\underline{\mathbb{L} = \{134\}}$

e) $\left(\frac{1}{2}m-1\right)^{-2/3} = 4 \quad \Big| \; \square^{-\frac{3}{2}}$

$$\frac{1}{2}m-1 = 4^{-3/2} \quad \Big| +1$$

$$\frac{1}{2}m = 4^{-3/2}+1 \quad \Big| \cdot 2$$

$$m = 2\cdot\left(\underbrace{4^{-3/2}}+1\right) = 2\cdot\left(\frac{1}{8}+1\right) = 2\cdot\frac{9}{8} = \frac{9}{4}$$

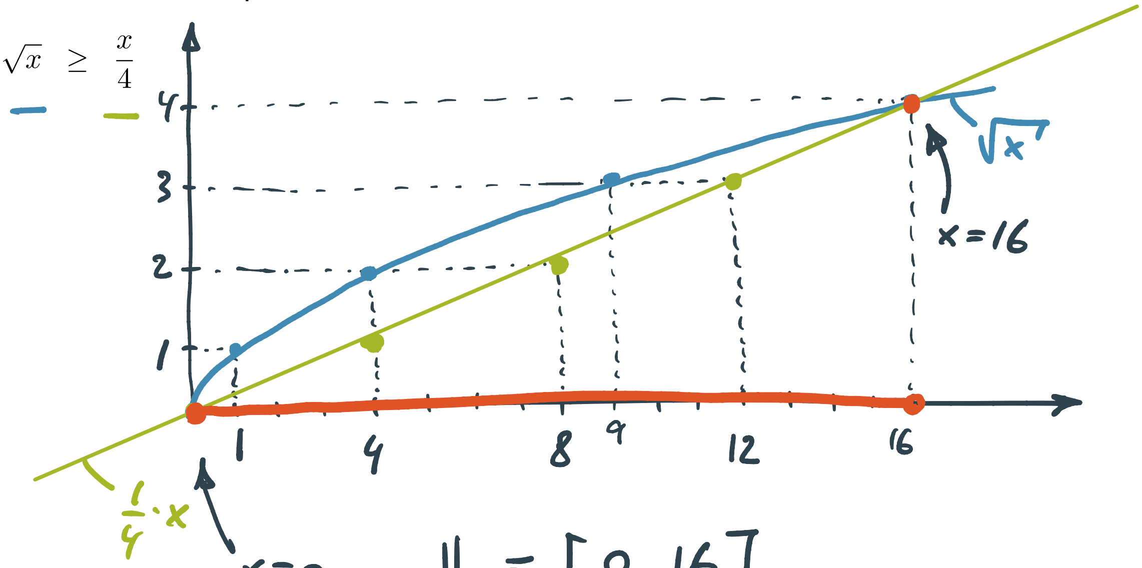
$$\left(\frac{1}{4}\right)^{3/2} = \frac{1}{4^{3/2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$4^{\frac{1}{2}\cdot 3} = \left(4^{\frac{1}{2}}\right)^3$$

$\underline{\mathbb{L} = \left\{\frac{1}{8}\right\}}$

Aufgabe 3 Finde die Lösungen der folgenden Ungleichungen mit Hilfe der grafischen Methode auf dem Computer.

a) $\sqrt{x} \quad \geq \quad \frac{x}{4}$

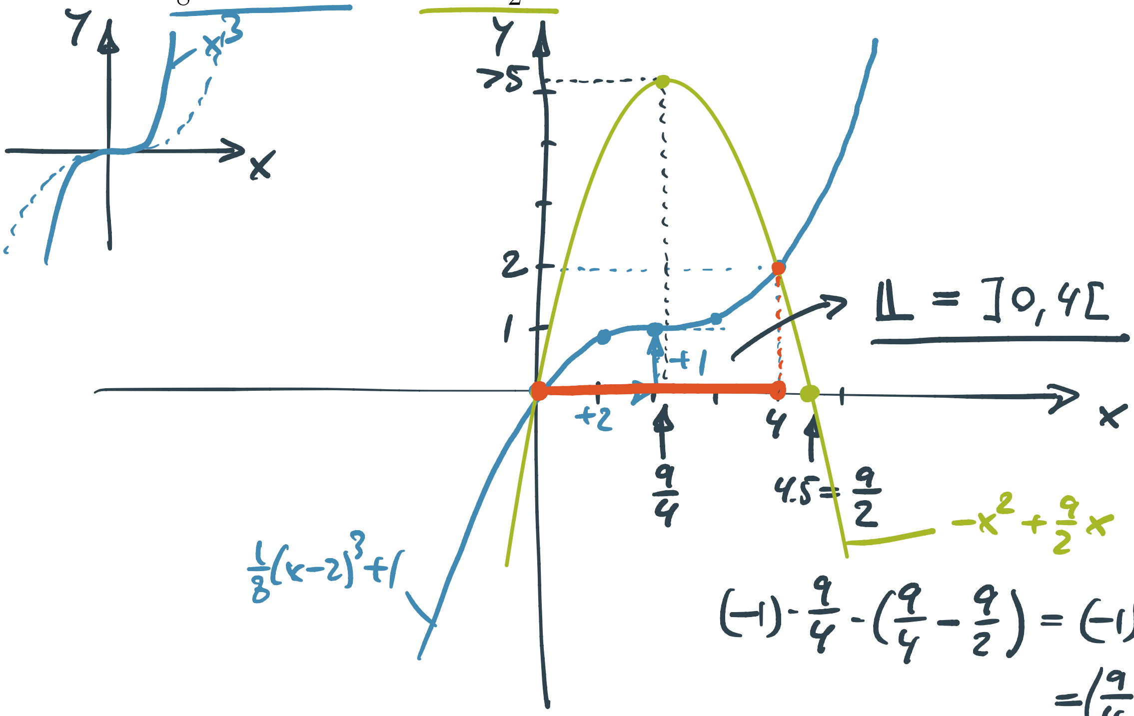


$\mathbb{L} = [0, 16]$

$$-x^2 + \frac{9}{2}x$$

$$(-1)\cdot x \cdot \left(x - \frac{9}{2}\right)$$

b) $\frac{1}{8}(x-2)^3 + 1 \quad < \quad -x^2 + \frac{9}{2}x$



$$\mathbb{L} =]0,4[$$

$$(-1)\cdot\frac{9}{4}\cdot\left(\frac{9}{4} - \frac{9}{2}\right) = (-1)\cdot\frac{9}{4}\cdot\left(-\frac{9}{4}\right)$$

$$= \left(\frac{9}{4}\right)^2 = \frac{81}{16} > 5$$

Aufgabe 4 Löse die folgenden Ungleichungen algebraisch. Achte darauf, dass das Ungleichheitszeichen bei gewissen Umformungen die Richtung ändert.

a) $\frac{2}{3}x < \frac{-x-2}{-3} \quad \vert \cdot 3$

$$2x < \frac{(-x-2)\cdot(-1)}{(-1)\cdot(-1)} = \frac{x+2}{1}$$

$$2x < x+2 \quad \vert -x$$

$$x < 2$$

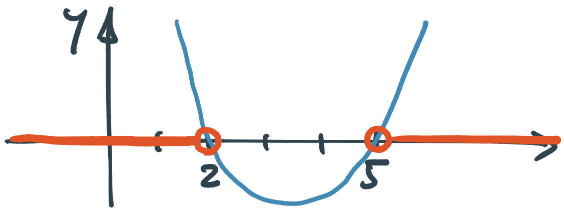
$$\underline{\mathbb{L} =]-\infty, 2[}$$

b) $x^2 - 4x + 4 \quad > \quad 3(x-2) \;=\; 3x - 6 \qquad | -3x$

$x^2 - 7x + 4 \quad > \quad -6 \qquad | +6$

$x^2 - 7x + 10 \quad > \quad 0$

$(x-2)\cdot(x-5) \quad > \quad 0$



$\underline{\mathbb{L} = \,]-\infty, 2[\, \cup \,]5, \infty[}$

oder: $\underline{\mathbb{L} = \mathbb{R} \setminus [2, 5]}$

c) $\frac{x}{x+1} \geq \frac{4(x-1)}{x^2-1} = \frac{4\cdot(\cancel{x-1})}{(x+1)(\cancel{x-1})}$

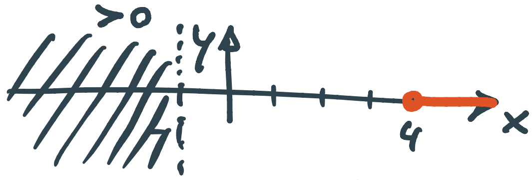
Fall 1

$x+1>0$

$x>-1$

$$\frac{x}{x+1} \geq \frac{4}{x+1} \quad \Big| \cdot \underbrace{(x+1)}_{>0}$$

$$x \geq 4$$



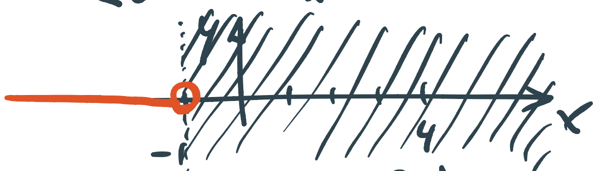
Fall 2

$x+1<0$

$x<-1$

$$\frac{x}{x+1} \geq \frac{4}{x+1} \quad \Big| \cdot \underbrace{(x+1)}_{<0}$$

$$x \leq 4$$



$\underline{\mathbb{L} = [4,\infty[\;\cup\;]-\infty,-1[}$ oder $\underline{\mathbb{L} = \mathbb{R} \setminus [-1,4[}$

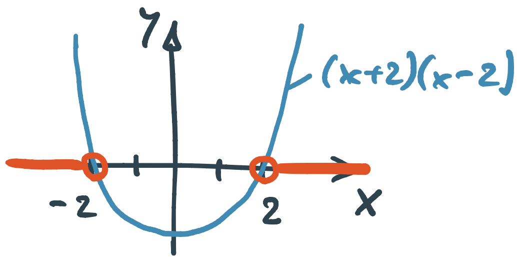
Aufgabe 5 Bestimme für die folgenden Ungleichungen die Lösungsmenge algebraisch.

a) $4 - x^2 < 0$

$(-1)\cdot(x^2-4) < 0 \quad | \cdot(-1)$

$x^2 - 4 > 0$

$(x+2)(x-2) > 0$



$\mathbb{L} =]-\infty, -2[\cup]2, \infty[$

oder $\mathbb{L} = \mathbb{R} \setminus [-2, 2]$

b) $\frac{x}{\sqrt{x^2+1}} \geq \frac{1}{2} \qquad \Big| \cdot \underbrace{\sqrt{x^2+1}}_{>0}$

$$x \geq \frac{1}{2}\sqrt{x^2+1}$$

$$x = \frac{1}{2}\sqrt{x^2+1} \qquad \Big| \,\square^2$$

$$x^2 = \frac{1}{4}\cdot(x^2+1)$$

$$= \frac{1}{4}x^2 + \frac{1}{4} \qquad \Big| -\frac{1}{4}x^2$$

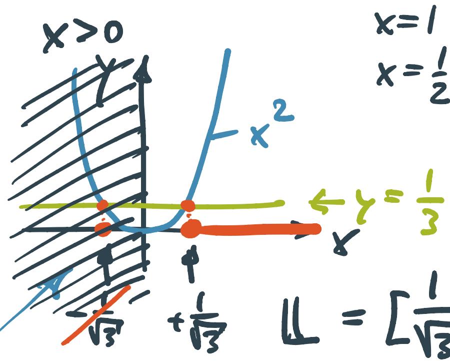
$$\frac{3}{4}x^2 = \frac{1}{4} \qquad \Big| \cdot 4$$

$$3x^2 = 1 \qquad \Big| :3$$

$$x^2 = \frac{1}{3} \qquad \Big| \sqrt{} \quad \rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$$

$x = \frac{1}{\sqrt{3}}$ ✓

$\cancel{x = -\frac{1}{\sqrt{3}}}$ Scheinlösung



$x = 1$ ✓

$x = \frac{1}{2}$ ✗

$$\mathbb{L} = \left[\frac{1}{\sqrt{3}}, \infty\right[$$

c) $12m > m^3 + m^2 \quad | :m$

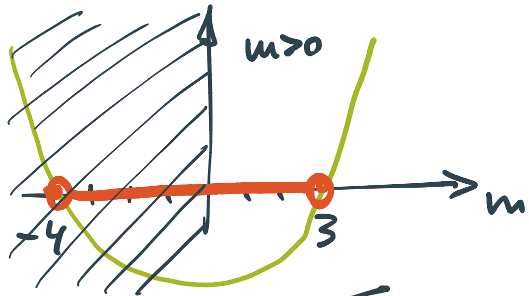
Fall 1

$m > 0$

$12 > m^2 + m \quad | -12$

$m^2 + m - 12 < 0$

$(m-3)\cdot(m+4) < 0$

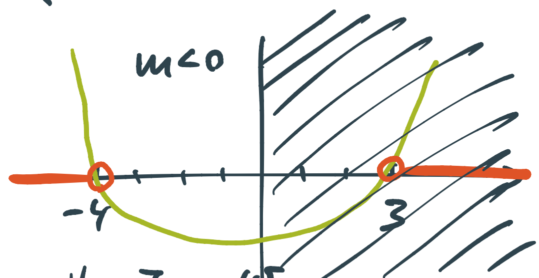


Fall 2

$m < 0$

$m^2 + m - 12 > 0$

$(m-3)(m+4) > 0$



$\cancel{\mathbb{L} =]-4, 3[}$ $\quad \mathbb{L} =]0, 3[\quad \mathbb{L} =]-\infty, -4[$

$\underline{\mathbb{L} =]-\infty, -4[\cup]0, 3[}$