Wurzelwerte

Aufgabe 1 Bringe die folgenden Terme in Normalform.

a) $\frac{1}{\sqrt{5}} = \frac{1 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{5}}{5} = \underline{\frac{1}{5}\sqrt{5}}$

b) $\sqrt{72} = \sqrt{2 \cdot 36} = \sqrt{2} \cdot \sqrt{36} = \underline{6\sqrt{2}}$

c) $\frac{6\sqrt{2}}{\sqrt{6}} = \frac{6\sqrt{2} \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{\cancel{6}\sqrt{12}}{\cancel{6}} = \sqrt{12} = \sqrt{3 \cdot 4} = \sqrt{3} \cdot \sqrt{4} = \underline{2\sqrt{3}}$

d) $\frac{\sqrt{3}}{1+\sqrt{2}} = \frac{\sqrt{3} \cdot (1-\sqrt{2})}{(1+\sqrt{2}) \cdot (1-\sqrt{2})} = \frac{\sqrt{3} - \sqrt{3}\sqrt{2}}{1-2} = \sqrt{3}\sqrt{2} - \sqrt{3}$
$= \underline{\sqrt{6} - \sqrt{3}}$

e) $\frac{\sqrt{13}}{\sqrt{7}-\sqrt{11}} = \frac{\sqrt{13} \cdot (\sqrt{7}+\sqrt{11})}{(\sqrt{7}-\sqrt{11}) \cdot (\sqrt{7}+\sqrt{11})}$

$= \frac{\sqrt{13}\sqrt{7} + \sqrt{13}\sqrt{11}}{7-11} = \left(-\frac{1}{4}\right) \cdot \left(\sqrt{91} + \sqrt{143}\right)$
$= \underline{-\frac{1}{4}\sqrt{91} - \frac{1}{4}\sqrt{143}}$

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zu d): $(a+b)(a-b) = a^2 - b^2$ (binomische Formel)

Aufgabe 2 Bringe die folgenden Terme in Normalform.

a) $\frac{1}{\sqrt{ab^2}} = \frac{1\cdot\sqrt{ab^2}}{\sqrt{ab^2}\cdot\sqrt{ab^2}} = \frac{1}{ab^2}\cdot\sqrt{ab^2} = \frac{1}{ab^2}\cdot\sqrt{a}\cdot\underbrace{\sqrt{b^2}}_{b} = \frac{\cancel{b}}{ab^{\cancel{2}}}\sqrt{a} = \frac{1}{ab}\sqrt{a}$

b) $\sqrt{k(k+l)^2}$

c) $\frac{(a+b)\cdot\sqrt{c}}{\sqrt{a+b}}$

d) $\frac{\sqrt{c}}{\sqrt{a}+b}$

e) $\frac{x+y^4+2\sqrt{x}y^2}{\sqrt{x}+y^2}$

b) $\sqrt{k(k+l)^2} = \sqrt{k}\cdot\underbrace{\sqrt{(k+l)^2}}_{k+l} = \sqrt{k}\cdot(k+l)$
$= (k+l)\cdot\sqrt{k}$

c) $\frac{(a+b)\cdot\sqrt{c}\cdot\sqrt{a+b}}{\sqrt{a+b}\cdot\sqrt{a+b}} = \frac{\cancel{a+b}}{\cancel{a+b}}\cdot\sqrt{c}\sqrt{a+b}$
$= \sqrt{c(a+b)}$
$= \sqrt{ac+bc}$

d) $\frac{\sqrt{c}\cdot(\sqrt{a}-b)}{(\sqrt{a}+b)\cdot(\sqrt{a}-b)}$
$= \frac{\sqrt{ac}-b\sqrt{c}}{a-b^2} = \frac{1}{a-b^2}\sqrt{ac} - \frac{b}{a-b^2}\sqrt{c}$

e) $\frac{x+y^4+2\sqrt{x}y^2}{\sqrt{x}+y^2} = \frac{(\sqrt{x})^2+2\sqrt{x}y^2+(y^2)^2}{\sqrt{x}+y^2}$

$= \frac{(\sqrt{x}+y^2)^{\cancel{2}}}{\cancel{\sqrt{x}+y^2}} = \sqrt{x}+y^2$

Aufgabe 3 Bringe die folgenden Terme in Normalform.

a) $(\sqrt{3}+\sqrt{8})^2$

b) $(1+\sqrt{3}-\sqrt{5})^2$

c) $\dfrac{1+2\sqrt{2}}{3\sqrt{2}+5}$

d) $\dfrac{1+\sqrt{3}}{1-\sqrt{3}} - \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

a)
$$(\sqrt{3}+\sqrt{8})^2 = (\sqrt{3})^2 + 2\sqrt{3}\sqrt{8} + (\sqrt{8})^2$$
$$= 3 + 2\sqrt{24} + 8$$
$$= 11 + 2\underbrace{\sqrt{4}}_{2}\cdot\sqrt{6}$$
$$= \underline{11 + 4\sqrt{6}}$$

b)
$$(\underbrace{1+\sqrt{3}}_{a} - \underbrace{\sqrt{5}}_{b})^2$$
$$= (1+\sqrt{3})^2 - 2(1+\sqrt{3})\sqrt{5} + 5$$
$$= 1 + 2\cdot 1\cdot\sqrt{3} + 3 - 2(\sqrt{5}+\sqrt{15}) + 5$$
$$= \underline{9 + 2\sqrt{3} - 2\sqrt{5} - 2\sqrt{15}}$$

c)
$$\frac{(1+2\sqrt{2})\cdot(3\sqrt{2}-5)}{(3\sqrt{2}+5)\cdot(3\sqrt{2}-5)} = \frac{3\sqrt{2}-5+2\sqrt{2}\cdot 3\sqrt{2}-2\sqrt{2}\,5}{9\cdot 2 - 25}$$
$$= \frac{3\sqrt{2}-5+12-10\sqrt{2}}{-7} = \left(-\frac{1}{7}\right)\cdot(-7\sqrt{2}+7)$$
$$= -\cancel{\frac{7}{7}}\cdot(-\sqrt{2}+1) = \underline{\sqrt{2}-1}$$

d)
$$\frac{1+\sqrt{3}}{1-\sqrt{3}} - \frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} - \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$
$$= \frac{(1+\sqrt{3})^2-(1-\sqrt{3})^2}{1-3}$$
$$= \left(-\frac{1}{2}\right)\cdot\left((1+2\sqrt{3}+3)-(1-2\sqrt{3}+3)\right)$$
$$= \left(-\frac{1}{2}\right)\cdot\left(4+2\sqrt{3}-(4-2\sqrt{3})\right) = \frac{1}{2}\cdot\left(\cancel{4}-2\sqrt{3}-\cancel{4}-2\sqrt{3}\right) = -\frac{4}{2}\sqrt{3} = \underline{-2\sqrt{3}}$$

Aufgabe 4 Finde für die Unbekannte alle Lösungen, die die Gleichung erfüllen.

a) $\frac{2+\sqrt{x}}{3} = 5$

b) $6\sqrt[3]{a} = 2\frac{1}{2} + a^{\frac{1}{3}}$

c) $(v+1)^{\frac{2}{3}} = \frac{4}{4^{\frac{1}{3}}}$

d) $a\sqrt{6} + 1 = a + \sqrt{6}$

e) $(x+\sqrt{2})^2 = x^2 - x - 6$

a) $\frac{2+\sqrt{x}}{3} = 5 \quad | \cdot 3$

$2+\sqrt{x} = 15 \quad | -2$

$\sqrt{x} = 13 \quad |$ quadrieren

$\underline{x = 13^2 = 169}$

b) $6\sqrt[3]{a} = 2\frac{1}{2} + \sqrt[3]{a} \quad | -\sqrt[3]{a}$

$5\sqrt[3]{a} = \frac{3}{2} \quad | :5$

$\sqrt[3]{a} = \frac{3}{10} \quad |$ hoch 3

$a = \left(\frac{3}{10}\right)^3 = \frac{3^3}{10^3} = \frac{27}{1000} = \underline{0.027}$

c) $(v+1)^{\frac{2}{3}} = \frac{4^1}{4^{\frac{1}{3}}} = 4^{1-\frac{1}{3}} = 4^{\frac{2}{3}}$

$(v+1)^{\frac{2}{3}} = 4^{\frac{2}{3}} \quad |$ hoch 3

$(v+1)^2 = 4^2 = 16 \quad | \sqrt{\ }$

$v+1 = 4 \rightarrow \underline{v = 3}$

$(v+1)^2 = (-4)^2 = 16 \quad | \sqrt{\ }$

$v+1 = -4 \quad | -1$

$\underline{v = -5}$

$\Rightarrow \underline{\mathbb{L} = \{-5, 3\}}$

d) $a\sqrt{6} + 1 = a + \sqrt{6} \quad | -a$

$a\sqrt{6} - a + 1 = \sqrt{6} \quad | -1$

$a(\sqrt{6} - 1) = \sqrt{6} - 1 \quad | :(\sqrt{6}-1)$

$a = \frac{\sqrt{6}-1}{\sqrt{6}-1} = 1$

$\rightarrow \underline{a = 1}$

e) $(x+\sqrt{2})^2 = x^2 - x - 6$

$\cancel{x^2} + 2\sqrt{2}x + 2 = \cancel{x^2} - x - 6 \quad | -x^2 \; +x \; -2$

$2\sqrt{2}x + x = -8$

$x \cdot (2\sqrt{2} + 1) = -8 \quad | :(2\sqrt{2}+1)$

$x = -\frac{8 \cdot (2\sqrt{2}-1)}{(2\sqrt{2}+1)(2\sqrt{2}-1)} = -\frac{16\sqrt{2} - 8}{8 - 1}$

$x = -\frac{16}{7}\sqrt{2} + \frac{8}{7} = \underline{\frac{8}{7} - \frac{16}{7}\sqrt{2}}$

Aufgabe 5 Finde für die Unbekannte alle Lösungen, die die Gleichung erfüllen.

a) $\sqrt{\frac{4}{3}+a^2} = \frac{4}{3}$

b) $\sqrt{2-x}-\sqrt{x+1}=0$

c) $\sqrt{5b} = \sqrt{5+b}$

d) $\sqrt{3+a^2-2\sqrt{3}a} = a$

e) $2\sqrt{a} = \sqrt{a-3}+\sqrt{a-5}$

a) $\sqrt{\frac{4}{3}+a^2} = \frac{4}{3}$ | quadr.

$\frac{4}{3}+a^2 = \frac{16}{9}$ | $-\frac{4}{3}$

$a^2 = \frac{16}{9}-\frac{4}{3} = \frac{16-12}{9} = \frac{4}{9}$

$a^2 = \frac{4}{9}$ | $\sqrt{}$

$a = \frac{2}{3}$

$a = -\frac{2}{3}$

$\mathbb{L} = \{\pm\frac{2}{3}\}$

b) $\sqrt{2-x}-\sqrt{x+1}=0$ | $+\sqrt{x+1}$

$\sqrt{2-x} = \sqrt{x+1}$ | quadrieren

$2-x = x+1$ | $+x$ -1

$1 = 2x$

$x = \frac{1}{2}$

c) $\sqrt{5b} = \sqrt{5+b}$ | quadr.

$5b = 5+b$ | $-b$

$4b = 5$ | $:4$

$b = \frac{5}{4}$

d) $\sqrt{3+a^2-2\sqrt{3}a} = a$

$\sqrt{a^2-2a\sqrt{3}+(\sqrt{3})^2} = a$

$\sqrt{(a-\sqrt{3})^2} = a$

$a-\sqrt{3} = a$ | $-a$

$\cancel{-\sqrt{3} = 0}$

$[-(a-\sqrt{3})] = a$

$-a+\sqrt{3} = a$ | $+a$

$\sqrt{3} = 2a \rightarrow a = \frac{1}{2}\sqrt{3}$

a muss positiv sein!

e) $2\sqrt{a} = \underbrace{\sqrt{a-3}}_{A}+\underbrace{\sqrt{a-5}}_{B}$ | quadr.

$4a = \underbrace{(a-3)}_{A^2}+\underbrace{2\sqrt{a-3}\sqrt{a-5}}_{2AB}+\underbrace{(a-5)}_{B^2}$ | $-2a$ $+8$

$2a+8 = 2\sqrt{a^2-8a+15}$ | $:2$

$a+4 = \sqrt{a^2-8a+15}$ | quadrieren

$\cancel{a^2}+8a+16 = \cancel{a^2}-8a+15$

$16a = -1$

$a = -\frac{1}{16} < 0$

$\Rightarrow$ keine Lösung!