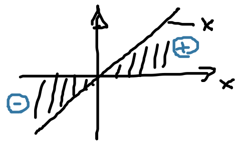
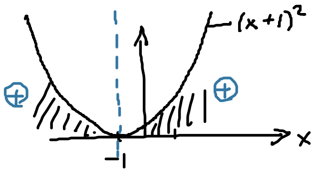
**Aufgabe 1:** Finden Sie den Wert der folgenden Integrale:

a) $\int_{-\infty}^{\infty} x\,dx = \underline{0}$

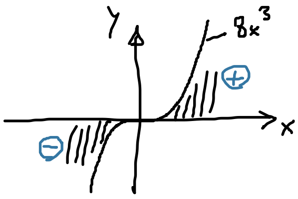



ungerade Funktion

b) $\int_{-\infty}^{\infty} \underbrace{(x+1)^2}_{u}\,dx \to \underline{\infty}$

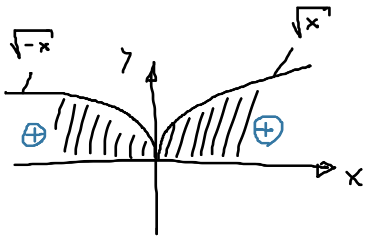



c) $\int_{-\infty}^{\infty} 8x^3\,dx = \underline{0}$




ungerade Funktion

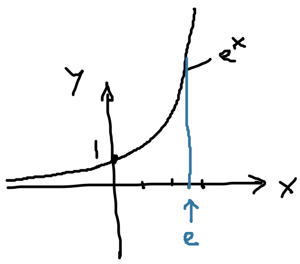
d) $\int_{-\infty}^{\infty} \sqrt{|x|}\,dx \to \underline{\infty}$




$x > 0: \quad f(x) = \sqrt{x}$

$x < 0: \quad f(x) = \sqrt{-x}$

e) $\int_{e}^{e} e^x\,dx = \underline{0}$



**Aufgabe 2:** Berechnen Sie die folgenden Integrale:

a) $\int_1^\infty e^{-4x}\,dx = \left[-\frac{1}{4}e^{-4x}\right]_1^\infty = \lim_{b\to\infty}\left(-\frac{1}{4}e^{-4b}\right) - \left(-\frac{1}{4}e^{-4\cdot 1}\right)$

$= -\frac{1}{4}\underbrace{\lim_{b\to\infty}\left(\underbrace{e^{-4b}}_{\frac{1}{e^{4b}}}\right)}_{\to 0} + \frac{1}{4}e^{-4} = \frac{1}{4}e^{-4} = \underline{\frac{1}{4e^4}}$

b) $\int_1^\infty \frac{1}{\sqrt{t}}\,dt = \int_1^\infty t^{-1/2}\,dt = \left[2\cdot t^{1/2}\right]_1^\infty = 2\cdot\underbrace{\lim_{b\to\infty}\left(b^{1/2}\right)}_{\to\infty} - 2\cdot 1^{1/2} \underline{\to \infty}$

c) $\int_{-\infty}^{-2} x^{-3}\,dx = \left[-\frac{1}{2}x^{-2}\right]_{-\infty}^{-2} = -\frac{1}{2}(-2)^{-2} - \lim_{a\to-\infty}\left(-\frac{1}{2}a^{-2}\right)$

$= -\frac{1}{8} + \frac{1}{2}\underbrace{\lim_{a\to-\infty}\left(\frac{1}{a^2}\right)}_{\to 0} = \underline{-\frac{1}{8}}$

d) $\int_{-1}^\infty \frac{m-7}{m^2-49}\,dm = \int_{-1}^\infty \frac{\cancel{m-7}}{(m+7)\cancel{(m-7)}}\,dm = \int_{-1}^\infty \frac{1}{m+7}\,dm = \left[\ln|m+7|\right]_{-1}^\infty$

$= \underbrace{\lim_{b\to\infty}\left(\ln|b+7|\right)}_{\to\infty} - \ln|-1+7| \underline{\to \infty}$

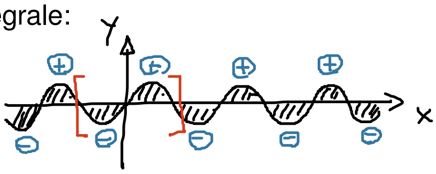
e) $\int_{-\infty}^0 x^2\cdot e^{x^3}\,dx = \left[\frac{1}{3}e^{x^3}\right]_{-\infty}^0 = \frac{1}{3}e^{0^3} - \lim_{a\to-\infty}\left(\frac{1}{3}e^{a^3}\right)$

$= \frac{1}{3} - \underbrace{\frac{1}{3}\lim_{a\to-\infty}\left(e^{a^3}\right)}_{\to 0} = \underline{\frac{1}{3}}$

**Aufgabe 3:** Berechnen Sie die folgenden Integrale:

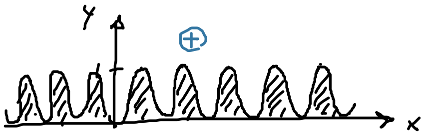
a) $\int_{-\infty}^{\infty} \sin(x)\, dx = \underline{0}$

ungerade Funktion



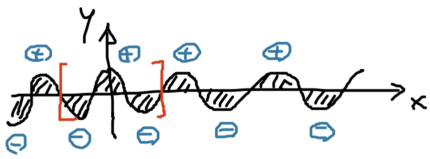

b) $\int_{-\infty}^{\infty} \sin^2(x)\, dx \quad \underline{\to \infty}$

gerade Funktion




c) $\int_{-\infty}^{\infty} \cos(x)\, dx = \underline{0}$

gerade Funktion

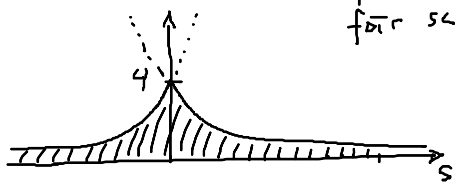



für $s>0$: $f(s) = 4e^{-4s}$

für $s<0$: $f(s) = 4e^{4s}$

d) $\int_{-\infty}^{\infty} 4e^{-4|s|}\, ds = 2 \cdot \int_{-\infty}^{0} 4e^{4s}\, ds$

gerade Funktion

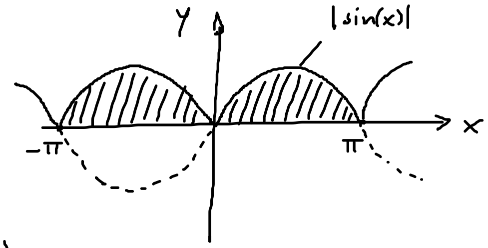



$$= 8 \cdot \int_{-\infty}^{0} e^{4s}\, ds = 8 \cdot \left[\frac{1}{4} e^{4s}\right]_{-\infty}^{0} = 8 \cdot \left(\frac{1}{4} e^{0} \underbrace{- \frac{1}{4} \lim_{a \to -\infty} \left(e^{4a}\right)}_{\to 0}\right) = 8 \cdot \left(\frac{1}{4} - 0\right)$$

$$= \underline{2}$$

**Aufgabe 4:** Berechnen Sie die folgenden Integrale und skizzieren Sie den Graphen und die Fläche, die sie darstellen:

a) $\int_{-\pi}^{\pi} |\sin(x)|\, dx$ → gerade Funktion




$$= 2\int_0^{\pi} \sin(x)\,dx = 2\cdot\left[-\cos(x)\right]_0^{\pi}$$

$$= 2\cdot\left((-\cos(\pi)) - (-\cos(0))\right)$$

$$= 2\cdot(+1+1) = \underline{4}$$

b) $\int_6^{10} \left(4 - 2\cdot|x-8|\right) dx$

für $(x-8)>0$
$\Leftrightarrow x > 8$  $f(x) = 4-2\cdot(x-8) = 4-2x+16 = \underline{20-2x}$

für $(x-8)<0$
$\Leftrightarrow x < 8$  $f(x) = 4-2\cdot\underbrace{[-(x-8)]}_{8-x} = 4-16+2x = \underline{-12+2x}$

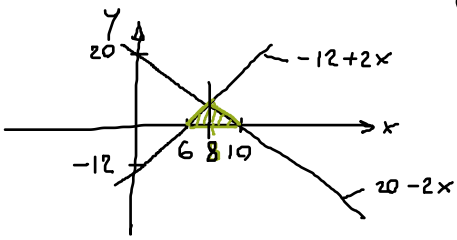
$$= \int_6^8(-12+2x)\,dx + \int_8^{10}(20-2x)\,dx$$

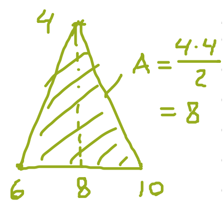
$$= \left[-12x+x^2\right]_6^8 + \left[20x-x^2\right]_8^{10}$$

$$= (-12\cdot 8+64)-(-12\cdot 6+36)+(20\cdot 10-100) - (20\cdot 8-64)$$

$$= -\cancel{96}+\cancel{64}+\cancel{72}-\cancel{36}+\cancel{200}-\cancel{100}-\cancel{160}+\cancel{64}$$

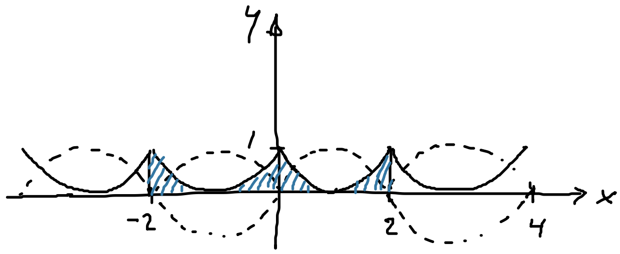
$$= 100+128+36-256 = 264-256 = \underline{8}$$




c) $\int_{-2}^{2} \left(1 - |\sin\left(\frac{\pi x}{2}\right)|\right) dx$

gerade Funktion




$$= 2\cdot\int_0^2 \left(1-\sin\left(\tfrac{\pi x}{2}\right)\right)dx$$

$$= 2\cdot\left[x + \tfrac{2}{\pi}\cdot\cos\tfrac{\pi x}{2}\right]_0^2$$

$$= 2\cdot\left(2+\tfrac{2}{\pi}\cos\pi - 0 - \tfrac{2}{\pi}\cos 0\right)$$

$$= 2\cdot\left(2+\tfrac{2}{\pi}(-1) - \tfrac{2}{\pi}\right)$$

$$= 2\cdot\left(2-\tfrac{4}{\pi}\right) = 4-\tfrac{8}{\pi} = \underline{4\left(1-\tfrac{2}{\pi}\right)}$$

$\sin(x)$ → Periode: $2\pi$
$\sin\left(\frac{x}{2}\right)$ → Periode: $4\pi$
$\sin\left(\frac{x}{2}\cdot\pi\right)$ → Periode: $4$