Aufgabe 1: Bestimmen Sie die Ableitungsfunktion $y'(x)$ mit Hilfe des Differentialquotienten für die folgenden Funktionen:

a) $y(x) = 4x^2 + 5x$ $\quad y(x+\Delta x) = 4\cdot(x+\Delta x)^2 + 5\cdot(x+\Delta x) = 4x^2 + 8x\Delta x + 4\Delta x^2 + 5x + 5\Delta x$

$$y'(x) = \lim_{\Delta x \to 0}\left(\frac{\cancel{4x^2} + 8x\Delta x + 4\cancel{\Delta x^2} + \cancel{5x} + 5\Delta x - \cancel{4x^2} - \cancel{5x}}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{8x\cancel{\Delta x} + 5\cancel{\Delta x}}{\cancel{\Delta x}}\right) = 8x + 5$$

$$\underline{y'(x) = 8x + 5}$$

b) $y(x) = \dfrac{1}{x^2}$ $\quad y(x+\Delta x) = \dfrac{1}{(x+\Delta x)^2}$ $\quad y'(x) = \lim\limits_{\Delta x \to 0}\left(\dfrac{\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}}{\Delta x}\right) = \lim\left(\dfrac{\frac{x^2 - (x+\Delta x)^2}{(x+\Delta x)^2 x^2}}{\Delta x}\right)$

$$y'(x) = \lim\left(\frac{\cancel{x^2} - \cancel{x^2} - 2x\cancel{\Delta x} - \cancel{\Delta x^2}}{(x+\Delta x)^2\, x^2\, \cancel{\Delta x}}\right) = \lim\left(-\frac{2\cancel{x}}{(x+\Delta x)^2 x^{\cancel{2}}}\right) = -\frac{2}{x^2 x} = -\frac{2}{x^3}$$

$$x^2 + 2\cancel{x\Delta x} + \cancel{\Delta x^2}$$

$$\rightarrow \underline{y'(x) = -\frac{2}{x^3}}$$

c) $y(x) = (3x-1)^2 = 9x^2 - 6x + 1$

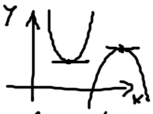
$$y(x+\Delta x) = 9(x+\Delta x)^2 - 6(x+\Delta x) + 1$$
$$= 9x^2 + 18x\Delta x + 9\Delta x^2 - 6x - 6\Delta x + 1$$

$$y'(x) = \lim_{\Delta x \to 0}\left(\frac{\cancel{9x^2} + 18x\cancel{\Delta x} + 9\cancel{\Delta x^2} - \cancel{6x} - 6\cancel{\Delta x} + \cancel{1} - \cancel{9x^2} + \cancel{6x} - \cancel{1}}{\cancel{\Delta x}}\right) = 18x - 6$$

$$\underline{y'(x) = 18x - 6}$$

Aufgabe 2: Geben Sie die Koordinaten der Punkte an, in welchen die gegebene Funktion horizontal ist.

a) $y(x) = 2x^2 - 5x$



$$y(x+\Delta x) = 2(x+\Delta x)^2 - 5(x+\Delta x) = 2x^2 + 4x\Delta x + 2\Delta x^2 - 5x - 5\Delta x$$

$$y'(x) = \lim_{\Delta x \to 0} \left(\frac{\cancel{2x^2} + 4x\cancel{\Delta x} + 2\cancel{\Delta x^2} - \cancel{5x} - 5\cancel{\Delta x} - \cancel{2x^2} + \cancel{5x}}{\cancel{\Delta x}} \right) = 4x - 5$$

$$y'(x_0) \overset{!}{=} 0 \qquad 4x_0 - 5 \overset{!}{=} 0$$

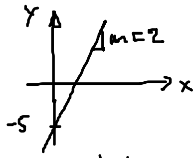
$$\rightarrow 4x_0 = 5 \rightarrow \underline{x_0 = \frac{5}{4}}$$

$$y_0 = 2x_0^2 - 5x_0 = 2\left(\frac{5}{4}\right)^2 - 5\cdot\frac{5}{4}$$

$$= \frac{25}{8} - \frac{25}{4} = 25\cdot\underbrace{\left(\frac{1}{8} - \frac{1}{4}\right)}_{-\frac{1}{8}}$$

$$\underline{y_0 = -\frac{25}{8}}$$

b) $y(x) = 2x - 5$



$y(x)$ ist <u>nie</u> horizontal $\rightarrow$ keine Lösung

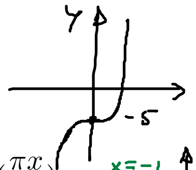
c) $y(x) = 2x^3 - 5$

$$y(x+\Delta x) = 2\cdot(x+\Delta x)^3 - 5 = 2x^3 + 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - 5$$

$$y'(x) = \lim_{\Delta x \to 0} \left(\frac{\cancel{2x^3} + 6x^2\cancel{\Delta x} + 6x\cancel{\Delta x^2} + 2\cancel{\Delta x^3} - \cancel{5} - \cancel{2x^3} + \cancel{5}}{\cancel{\Delta x}} \right) = 6x^2 \overset{!}{=} 0$$

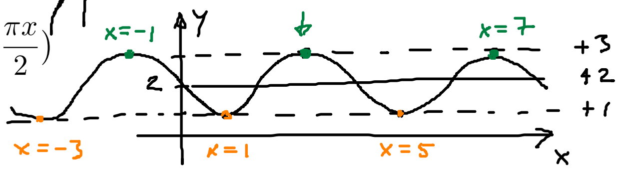
$$6x_0 = 0$$

$$\underline{y_0 = -5}$$



$$\frac{9\pi}{2} = u = \frac{\pi x}{2} \rightarrow x = 3$$

d) $y(x) = 2 - \sin\left(\frac{\pi x}{2}\right)$



$$\begin{cases} x_0 = 3 + 4n \quad n \in \mathbb{Z}, & y_0 = +3 \\ x_0 = 1 + 4n \quad n \in \mathbb{Z}, & y_0 = +1 \end{cases}$$

e) $y(x) = \left(\frac{x^2 - 9}{x - 3}\right)^2 + 1 = \left(\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}\right)^2 + 1 = x^2 + 6x + 9 + 1$

$$y(x) = x^2 + 6x + 10$$

$$y(x+\Delta x) = (x+\Delta x)^2 + 6(x+\Delta x) + 10 = x^2 + 2x\Delta x + \Delta x^2 + 6x + 6\Delta x + 10$$

$$y'(x) = \lim_{\Delta x \to 0} \left(\frac{\cancel{x^2} + 2x\cancel{\Delta x} + \cancel{\Delta x^2} + \cancel{6x} + 6\cancel{\Delta x} + \cancel{10} - \cancel{x^2} - \cancel{6x} - \cancel{10}}{\cancel{\Delta x}} \right) = 2x + 6$$

$$y'(x_0) = 2x_0 + 6 \overset{!}{=} 0 \rightarrow \underline{x_0 = -3}$$

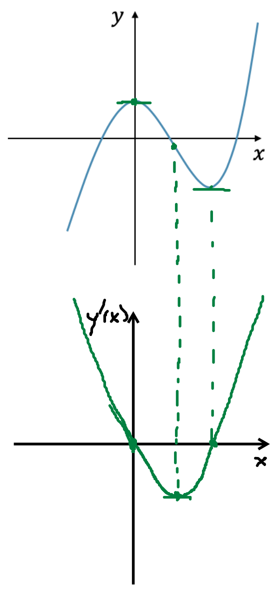
$$y_0 = (-3)^2 - 6\cdot 3 + 10$$

$$= 9 - 18 + 10 = 19 - 18 = 1$$

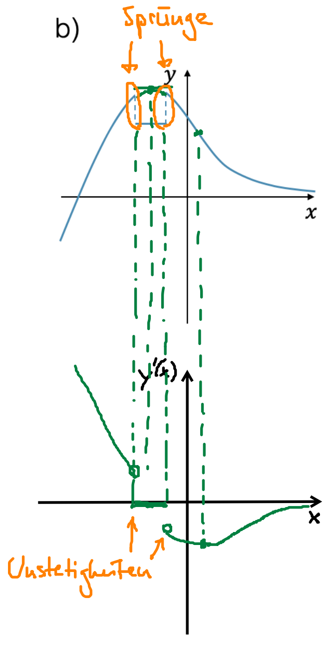
$$\underline{y_0 = 1}$$

Aufgabe 3: Skizzieren Sie den Verlauf der Ableitungsfunktion für die gegebenen Funktionsverläufe. Markieren Sie allfällige Stellen, in welchen die Ableitungsfunktion nicht definiert ist.

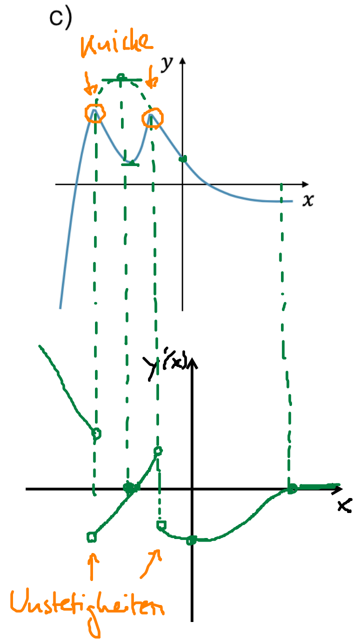
a)



b)

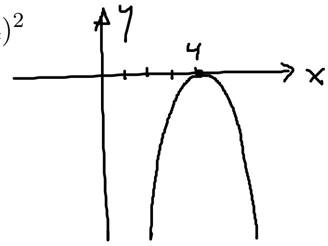


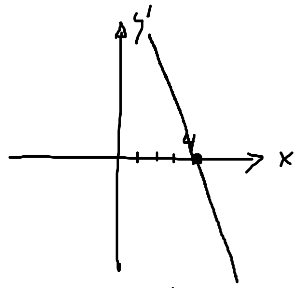
c)



Aufgabe 4: Skizzieren Sie den Verlauf der gegebenen Funktionen und markieren Sie die Stellen, an welchen die Funktion nicht differenzierbar ist, falls vorhanden. Skizzieren Sie dann den Verlauf der Ableitungsfunktion.

a) $y(x) = -(x-4)^2$



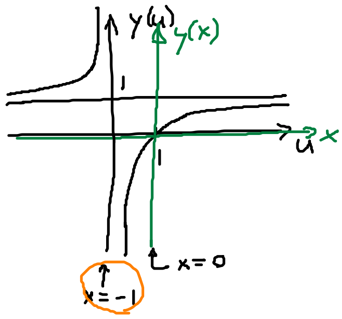


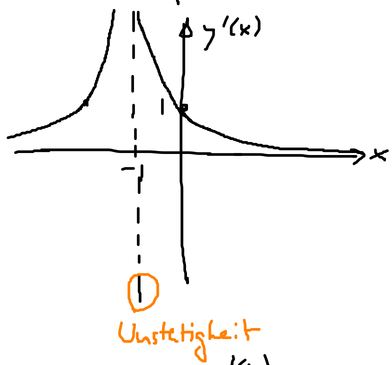
b) $y(x) = \dfrac{x}{x+1}$

Substitution: $u = x+1$

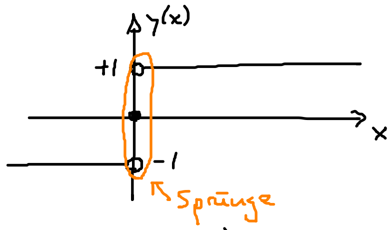
$\hookrightarrow x = u-1$

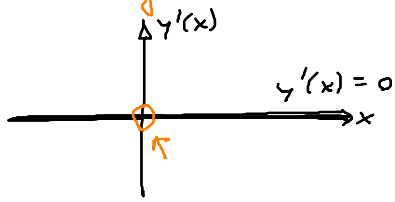
$y(u) = \dfrac{u-1}{u} = 1 - \dfrac{1}{u}$





c) $y(x) = \operatorname{sgn}(x)$

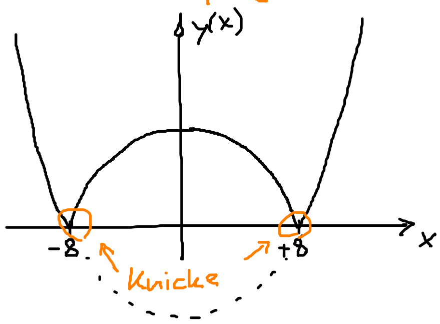


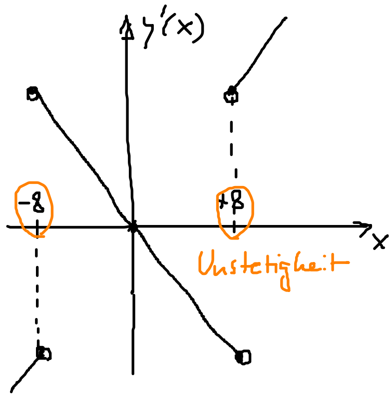


d) $y(x) = |-64 + x^2|$

$= |x^2 - 64|$

$= |(x-8)(x+8)|$





e) $y(x) = \dfrac{\sin(x)}{x}$

$= \dfrac{1}{x} \cdot \sin(x)$