Aufgabe 1: Berechnen Sie die Grenzwerte der Folgen.

a) $a_n = \dfrac{3n^2 + n}{4n^2}$

$$a = \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n^2+n}{4n^2} = \lim_{n\to\infty} \frac{3+\frac{1}{n}}{4} = \frac{\lim 3 + \lim_{n\to\infty} \frac{1}{n}}{\lim 4}$$

$$= \frac{3+0}{4} = \underline{\frac{3}{4}}$$

b) $a_n = \dfrac{(-1)^n - n}{n}$

$$a = \lim_{n\to\infty} \frac{(-1)^n - n}{n} = \lim \frac{\frac{(-1)^n}{n} - 1}{1} = \lim \frac{(-1)^n}{n} - \lim 1$$

$$= 0 - 1 = \underline{-1}$$

c) $a_k = \left(1 + \dfrac{1}{k^2}\right)^3$

$$\lim_{k\to\infty} \left[\left(1+\frac{1}{k^2}\right)^3\right] = \left[\lim_{k\to\infty}\left(1+\frac{1}{k^2}\right)\right]^3 = \Big[\underbrace{\lim 1}_{=1} + \underbrace{\cancel{\lim \frac{1}{k^2}}}_{=0}\Big]^3 = 1^3 = \underline{\underline{1}}$$

Lim ab = Lim a · Lim b

d) $b_n = n^{-2} \cdot ln(n+2) = \dfrac{\ln(n+2)}{n^2}$

$$b = \lim_{n\to\infty}\left(\frac{\ln(n+2)}{n^2}\right) = \underline{\underline{0}}$$

Dominanzregel
Log < Potenz

e) $a_j = \left(\dfrac{16}{8} - \dfrac{5}{4}\right)^j = \left(\dfrac{16-10}{8}\right)^j = \left(\dfrac{6}{8}\right)^j = \left(\dfrac{3}{4}\right)^j$

$$a = \lim_{j\to\infty} \Big(\underbrace{\frac{3}{4}}_{<1}\Big)^j = \underline{\underline{0}}$$

Aufgabe 2: Berechnen Sie die Grenzwerte der Folgen.

a) $a_n = \left(\frac{1}{n}+3\right)\left(1-\frac{1}{n}\right) \Rightarrow \lim a_n = \lim\left(\frac{1}{n}+3\right)\cdot\lim\left(1-\frac{1}{n}\right)$

$= \left[\underbrace{\lim_{n\to\infty}\left(\frac{1}{n}\right)}_{=0} + \underbrace{\lim 3}_{=3}\right]\cdot\left[\underbrace{\lim 1}_{=1} - \underbrace{\lim_{n\to\infty}\left(\frac{1}{n}\right)}_{=0}\right]$

$\lim(a\cdot b) = \lim a \cdot \lim b$

$\lim(a+b) = \lim a + \lim b$

$= 3\cdot 1 = \underline{\underline{3}}$

b) $a_n = \frac{n(n+1)^2}{n^3+2n^2+n} = \frac{\cancel{n}(n+1)^2}{\cancel{n}(n^2+2n+1)} = \frac{n^2+2n+1}{n^2+2n+1} = 1$

$\lim_{n\to\infty} a_n = \lim 1 = \underline{1}$

c) $b_n = \frac{2\sqrt{4n-1}}{\sqrt{n+1}}$ $\quad b_n^2 = \frac{4(4n-1)}{n+1} = \frac{16n-4}{n+1} = \frac{16-\frac{4}{n}}{1+\frac{1}{n}}$

$c = \lim_{n\to\infty} \underbrace{b_n^2}_{c_n} = \frac{\lim 16 - \cancel{\lim \frac{4}{n}}^{=0}}{\lim 1 + \underbrace{\cancel{\lim\frac{1}{n}}}_{=0}} = 16$

weil $b_n \geq 0 \Rightarrow b = \lim_{n\to\infty} b_n \geq 0$

$\Rightarrow \underline{b = +4}$

$c_n = b_n^2$

$\downarrow$

$c = 16 = b^2 \to b = \pm 4$

d) $c_j = \frac{2^j+1}{e^j} + 1$

$c_j = \left(\frac{2}{e}\right)^j + \left(\frac{1}{e}\right)^j + 1$ $\quad \lim_{j\to\infty} c_j = \underbrace{\lim\left(\frac{2}{e}\right)^j}_{} + \underbrace{\lim\left(\frac{1}{e}\right)^j}_{\Rightarrow 0} + \lim 1 = \underline{1}$

$e = 2.71$

$2 < e$

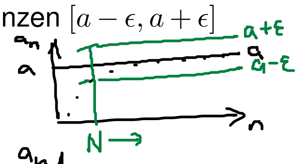
$\Rightarrow \left(\frac{2}{e}\right)^j < 1$

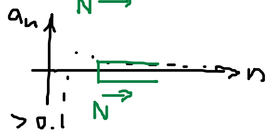
$\lim_{j\to\infty}\left(\frac{2}{e}\right)^j = 0$

e) $d_k = \frac{1+\frac{1}{k}}{2-\frac{1}{k}}$

$\lim_{k\to\infty} d_k = \frac{\lim 1 + \cancel{\lim\frac{1}{k}}^{=0}}{\lim 2 - \underbrace{\cancel{\lim\frac{1}{k}}}_{=0}} = \underline{\frac{1}{2}}$

Aufgabe 3: Bestimmen Sie, ab welchem Glied die Folgewerte innerhalb der Grenzen $[a-\epsilon, a+\epsilon]$ liegen.





a) $a_n = \frac{1}{2n-5}$ mit $\epsilon = 0.1$

$a = \lim_{n\to\infty} \frac{1}{2n-5} = \lim \frac{\frac{1}{n}}{2-\frac{5}{n}} = 0$

$\frac{1}{10} = 0.1 \overset{!}{=} \frac{1}{2N-5} \quad \Big| \cdot 2N-5$

$\frac{1}{10}(2N-5) = 1$

$\frac{2}{10}N - \frac{5}{10} = 1 \quad \Big| + \frac{5}{10}$

$\frac{2}{10}N = \frac{5}{10} + \frac{10}{10} = \frac{15}{10}$

$2N = 15$

$N = 7.5$

$a_7 = \frac{1}{14-5} = \frac{1}{9} = 0.111 > 0.1$

$\underline{\underline{\text{ab}}}$ $a_8 = \frac{1}{16-5} = \frac{1}{11} = 0.091 < 0.1$ ✓

b) $a_n = \frac{n-3}{n+3}$ mit $\epsilon = 0.03$

$a = \lim \frac{n-3}{n+3} = \frac{\lim 1 - \cancel{n}\frac{3}{n}}{\lim 1 + \cancel{n}\frac{3}{n}} = 1$

$a_3 = 0$

$a_4 = \frac{1}{7}$

$a_5 = \frac{1}{4}$

$0.97 \overset{!}{=} \frac{N-3}{N+3}$

$0.97N + 3 \cdot 0.97 = N - 3$

$3(1+0.97) = (1-0.97)N$

$N = \frac{3(1+0.97)}{1-0.97} = 197$ $\quad \underline{\underline{\text{ab } a_{197}}}$

$a_{197} = 0.97$

$a_{198} = 0.97015$

c) $a_k = \frac{k^2+k}{2k^2}$ mit $\epsilon = 10^{-3}$

$a = \lim_{k\to\infty} \frac{1+\frac{1}{k}}{2} = \frac{1}{2}$

$a_{10} = \frac{100+10}{200} = \frac{110}{200} > \frac{1}{2}$

$= \frac{550}{1000} = 0.500 + \underbrace{0.050}_{> 0.001}$

$0.501 \overset{!}{=} \frac{N^2+N}{2N^2} = \frac{\cancel{N}(N+1)}{\cancel{N}\cdot 2N} = \frac{N}{2N} + \frac{1}{2N}$

$0.501 = \frac{1}{2} + \frac{1}{2N}$

$0.001 = \frac{1}{2N}$

$1000 = 2N$

$N = 500$

$a_{500} = 0.501$

$a_{501} = 0.500998$

$\underline{\underline{\text{ab } a_{500}}}$

d) $a_n = \frac{2^{n+5}}{5^n}$ mit $\epsilon = 0.001$

$a = \lim_{n\to\infty} \frac{2^5 \cdot 2^n}{5^n} = \lim_{n\to\infty} 32 \cdot \underbrace{\left(\frac{2}{5}\right)^n}_{<1} = 0$

$0.001 \overset{!}{=} 32 \cdot \left(\frac{2}{5}\right)^N$

$\frac{1}{1000} = 32 \cdot \left(\frac{2}{5}\right)^N \quad \Big| : 32$

$\ln\left(\frac{1}{32'000}\right) = \ln\left(\frac{2}{5}\right)^N \quad \Big| \ln$

$-\ln(32'000) = N \cdot \ln\left(\frac{2}{5}\right)$

$N = \frac{-\ln(32'000)}{\ln\left(\frac{2}{5}\right)} = 11.3$

$a_{11} = 0.00134$

$\underline{\underline{\text{ab } a_{12}}} = 0.0005$ ✓

e) $a_n = \frac{n(n-4)}{(n-2)(n+3)}$ mit $\epsilon = \frac{1}{2}$

$a = \lim_{n\to\infty} \frac{n^2-4n}{n^2+n-6} = \lim_{n\to\infty} \frac{1-\cancel{\frac{4}{n}}}{1+\cancel{\frac{1}{n}}-\cancel{\frac{6}{n^2}}} = 1$

$a_{100} = 0.95$

$a_{10} = \frac{100-40}{100+10-6} = \frac{60}{104} < 0.6$

$a_5 = \frac{25-20}{25+5-6} = \frac{5}{24} > 0.2$

$0.5 \overset{!}{=} \frac{N^2-4N}{N^2+N-6}$

$N^2+N-6 = 2N^2-8N \quad \Big| -N^2-N+6$

$0 = N^2-9N+6$

$a=1, \; b=-9, \; c=6$

$D = 81-24 = 57 \rightarrow \sqrt{D} = \sqrt{57}$

$N = \frac{9 \pm \sqrt{57}}{2} = 4.5 \pm \frac{1}{2}\sqrt{57}$ $\begin{cases} \cancel{0.725} \\ 8.275 \end{cases}$

$a_8 = 0.4848$

$\underline{\underline{\text{ab } a_9}} = 0.536$ ✓