

Aufgabe 1: Bestimmen Sie die Lösungen der folgenden quadratischen Gleichungen.

$$\begin{array}{l}
 \text{a) } x^2 - 102 = -3x^2 + 42 \\
 4x^2 - 102 = 42 \\
 4x^2 = 144 \\
 x^2 = 36 \\
 x = \pm 6
 \end{array}
 \quad
 \begin{array}{l}
 | +3x^2 \\
 | +102 \\
 | :4 \\
 | \sqrt{}
 \end{array}$$

$$\begin{array}{l}
 \text{b) } 7(a^2 - 6) = 30 - a^2 \\
 7a^2 - 42 = 30 - a^2 \\
 8a^2 = 72 \\
 a^2 = 9 \\
 a = \pm 3
 \end{array}
 \quad
 \begin{array}{l}
 | +a^2 +42 \\
 | :8 \\
 | \sqrt{}
 \end{array}$$

$$\begin{array}{l}
 \text{c) } (z - 8)(z + 8) = 8^2 \\
 z^2 - 64 = 64 \\
 z^2 = 128 = 2^7 \\
 z = \pm 2^{3.5} = \pm (2^{3+0.5}) = \pm \left(\underbrace{2^3}_{8} \cdot \underbrace{2^{0.5}}_{\sqrt{2}} \right) = \pm 8\sqrt{2}
 \end{array}$$

$$\begin{array}{l}
 \text{d) } \frac{78}{39} + \frac{f^2}{3} = \frac{117}{39} \quad | \cdot 39 \\
 78 + \frac{39}{3} f^2 = 117 \\
 78 + 13f^2 = 117 \quad | -78 \\
 13f^2 = 39 \quad | :13
 \end{array}$$

$$\begin{array}{l}
 f^2 = 3 \\
 f = \pm \sqrt{3}
 \end{array}$$

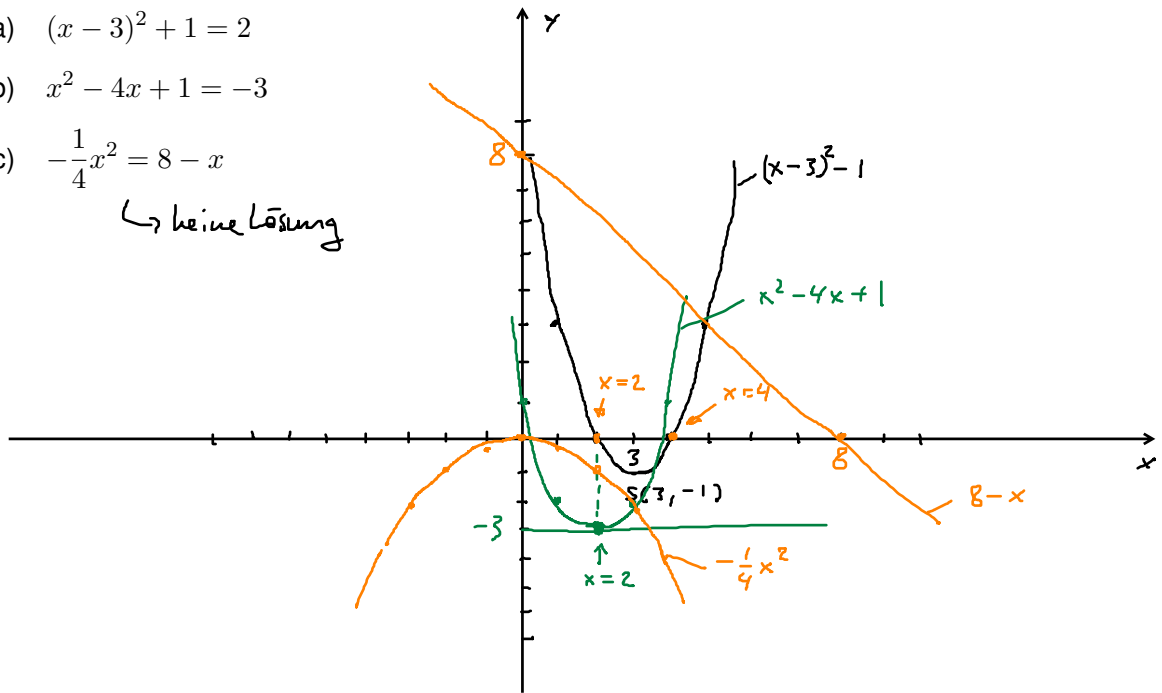
Aufgabe 2: Lösen Sie die folgenden Gleichungen mit Hilfe der grafischen Methode.

a) $(x-3)^2 + 1 = 2$

b) $x^2 - 4x + 1 = -3$

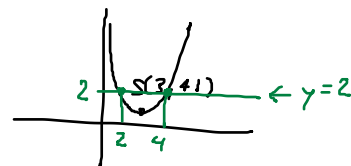
c) $-\frac{1}{4}x^2 = 8 - x$

↳ keine Lösung



a) $(x-3)^2 + 1 = 2 \quad | -2$
 $(x-3)^2 - 1 = 0 \rightarrow S(3, -1)$
 ↑
 (+1) positiver Vorfaktor

$\mathbb{L} = \{2, 4\}$



b) $x^2 - 4x + 1 = -3 \quad | +3$
 $x^2 - 4x + 4 = 0$
 $(x-2)^2 = 0 \rightarrow x=2 \rightarrow \mathbb{L} = \{2\}$

c) $-\frac{1}{4}x^2 = 8 - x \quad | \cdot 4$
 $-x^2 = 32 - 4x \quad | + x^2$
 $x^2 - 4x + 32 = 0$
 ↑ ↑ ↑
 $a=1 \quad b=-4 \quad c=32$

$D = b^2 - 4ac$
 $= 16 - 4 \cdot 1 \cdot 32 < 0$
 $D < 0 \Rightarrow$ keine reelle Lösung!

Aufgabe 3: Bestimmen Sie die Lösung mit Hilfe der allgemeinen Lösung für quadratische Gleichungen.

a) $x^2 + 14x + 33 = 0$
 $\uparrow \quad \uparrow \quad \uparrow$
 $a=1 \quad b=14 \quad c=33$

$$D = b^2 - 4ac = 14^2 - 4 \cdot 1 \cdot 33 = 196 - 132 = 64$$

$$\hookrightarrow \sqrt{D} = 8$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-14 \pm 8}{2} = -7 \pm 4 \quad \underline{\underline{L = \{-3, -11\}}}$$

b) $b^2 + 4b + 1 = 0$

$$A=1, \quad B=4, \quad C=1$$

$$D = B^2 - 4AC = 4^2 - 4 \cdot 1 \cdot 1 = 16 - 4 = 12$$

$$\sqrt{D} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

$$b = \frac{-B \pm \sqrt{D}}{2A} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3} \quad \underline{\underline{L = \{-2 - \sqrt{3}, -2 + \sqrt{3}\}}}$$

c) $3y^2 - 18 + 6y = 6y$ $\left| \begin{array}{l} -6y \\ +18 \end{array} \right.$

$$3y^2 - 18 = 0$$

$$3y^2 + 0y - 18 = 0 \quad a=3, b=0, c=-18$$

$$D = b^2 - 4ac = 0 + 4 \cdot 3 \cdot 18 = 216 = 4 \cdot 54 = 4 \cdot 9 \cdot 6$$

$$\sqrt{D} = \sqrt{4 \cdot 9 \cdot 6} = 2 \cdot 3 \cdot \sqrt{6} = 6\sqrt{6}$$

$$y = \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm 6\sqrt{6}}{2 \cdot 3} = \pm \sqrt{6} \rightarrow \underline{\underline{L = \{-\sqrt{6}, +\sqrt{6}\}}}$$

d) $4x^2 - 2\sqrt{2}x + 0.5 = 0$
 $a=4, \quad b=-2\sqrt{2}, \quad c=\frac{1}{2}$

$$D = b^2 - 4ac = 8 - 4 \cdot 4 \cdot \frac{1}{2} = 8 - 8 = 0 \rightarrow \sqrt{D} = 0$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{2\sqrt{2}}{8} = \frac{1}{4}\sqrt{2} \rightarrow \underline{\underline{L = \left\{ \frac{1}{4}\sqrt{2} \right\}}}$$

$$\frac{1}{4}\sqrt{2} = \sqrt{\frac{2}{16}} = \frac{1}{\sqrt{8}}$$

e) $4s(s-1) = 2s^2 - 3$ $\left| -2s^2 + 3 \right.$

$$4s^2 - 4s = 2s^2 - 3$$

$$2s^2 - 4s + 3 = 0$$

$$a=2, \quad b=-4, \quad c=3 \rightarrow D = b^2 - 4ac = 16 - 4 \cdot 2 \cdot 3 = 16 - 24 < 0$$

\rightarrow keine Lösung!

Aufgabe 4: Bestimmen Sie die Lösung mit Hilfe der Methode des quadratischen Ergänzens.

a) $x^2 - 4x = 0$

$$\begin{aligned} &\downarrow \\ &x^2 - 2 \cdot x \cdot 2 + 4 - 4 = 0 \\ &\underbrace{(x-2)^2 - 4 = 0}_{(x-2)^2 = 4} \quad \left| +4 \right. \\ &x - 2 = \pm 2 \end{aligned}$$

$$\begin{aligned} x - 2 &= \pm 2 \quad | +2 \\ x &= 2 \pm 2 \\ \hookrightarrow \underline{\mathbb{L}} &= \{4, 0\} \end{aligned}$$

b) $k^2 + 24k + 135 = 0$

$$\begin{aligned} &\downarrow \\ &k^2 + 2 \cdot k \cdot 12 + 144 - 144 + 135 = 0 \\ &\underbrace{(k+12)^2 - 9 = 0}_{(k+12)^2 = 9} \end{aligned}$$

$$\begin{aligned} k + 12 &= \pm 3 \quad | -12 \\ k &= -12 \pm 3 \\ \hookrightarrow \underline{\mathbb{L}} &= \{-9, -15\} \end{aligned}$$

c) $3a^2 - 27a + 40 = -2$

$$\begin{aligned} &3a^2 - 27a + 42 = 0 \quad \left| :3 \right. \\ &a^2 - 9a + 14 = 0 \\ &\downarrow \\ &a^2 - 2 \cdot a \cdot 4.5 + 4.5^2 - 4.5^2 + 14 = 0 \\ &\underbrace{(a-4.5)^2 - 6.25 = 0}_{(a-4.5)^2 = 6.25} \end{aligned}$$

$$\begin{aligned} \left(\frac{9}{2}\right)^2 &= \frac{81}{4} = 20.25 \\ (a-4.5)^2 &= 6.25 \\ a - 4.5 &= \pm 2.5 \\ a &= 4.5 \pm 2.5 \\ \hookrightarrow \underline{\mathbb{L}} &= \{7, 2\} \end{aligned}$$

d) $\frac{1}{2}x^2 + 5x = 0 \quad | \cdot 2$

$$\begin{aligned} &x^2 + 10x = 0 \\ &\downarrow \\ &x^2 + 2 \cdot x \cdot 5 + 5^2 - 5^2 = 0 \\ &\underbrace{(x+5)^2 - 25 = 0}_{(x+5)^2 = 25} \end{aligned}$$

$$\begin{aligned} (x+5)^2 &= 25 \\ x + 5 &= \pm 5 \\ x &= -5 \pm 5 \rightarrow \underline{\mathbb{L}} = \{0, -10\} \end{aligned}$$

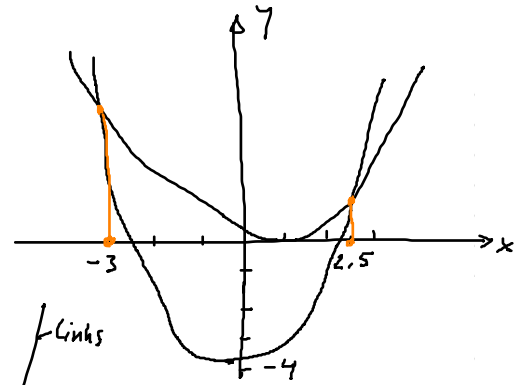
e) $z(z - 2\sqrt{2}) = 0$

$$\begin{aligned} &z^2 - 2\sqrt{2}z = 0 \\ &z^2 - 2 \cdot z \cdot \sqrt{2} + (\sqrt{2})^2 - (\sqrt{2})^2 = 0 \\ &\underbrace{(z - \sqrt{2})^2 - 2 = 0}_{(z - \sqrt{2})^2 = 2} \\ &z - \sqrt{2} = \pm \sqrt{2} \rightarrow z = \sqrt{2} \pm \sqrt{2} \rightarrow \underline{\mathbb{L}} = \{0, 2\sqrt{2}\} \end{aligned}$$

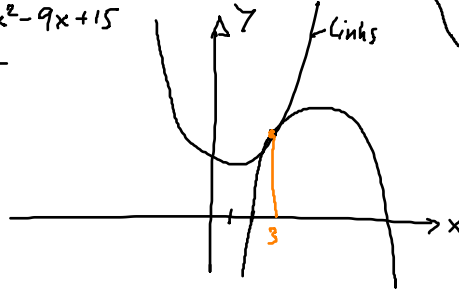
Aufgabe 5: Lösen Sie die folgenden Gleichungen zuerst von Hand mit der Methode Ihrer Wahl. Überprüfen Sie dann Ihr Resultat mit der grafischen Methode auf dem Computer.

a) $x^2 - \frac{15}{4} = \frac{2x^2 - x}{4} \quad | \cdot 4$

$4x^2 - 15 = 2x^2 - x \quad | -2x^2 + x$
 $2x^2 + x - 15 = 0 \rightarrow a=2, b=1, c=-15$
 $\Delta = b^2 - 4ac = 1 + 4 \cdot 2 \cdot 15 = 121$
 $\rightarrow \sqrt{\Delta} = 11$
 $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 11}{2 \cdot 2} = \frac{-1 \pm 11}{4} \rightarrow \mathbb{L} = \{-3, 2.5\}$



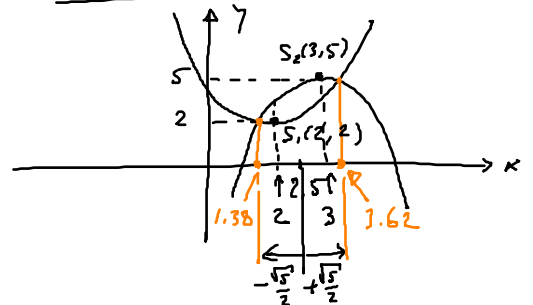
b) $x^2 - 3x + 3 = -x^2 + 9x - 15 \quad | +x^2 - 9x + 15$
 $2x^2 - 12x + 18 = 0 \quad | :2$
 $x^2 - 6x + 9 = 0$
 $(x-3)^2 = 0$
 $\hookrightarrow \mathbb{L} = \{3\}$



c) $(y-2)^2 + 2 = -(y-3)^2 + 5$

$y^2 - 4y + 4 + 2 = -(y^2 - 6y + 9) + 5$
 $y^2 - 4y + 6 = -y^2 + 6y - 9 + 5 \quad | +y^2 - 6y + 4$
 $2y^2 - 10y + 10 = 0 \quad | :2$
 $y^2 - 5y + 5 = 0$
 $a=1 \quad b=-5 \quad c=5$

$D = b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 5 = 25 - 20 = 5$
 $\sqrt{D} = \sqrt{5}$
 $y = \frac{-b \pm \sqrt{D}}{2a} = \frac{5 \pm \sqrt{5}}{2}$
 $\mathbb{L} = \left\{ \frac{5 + \sqrt{5}}{2}, \frac{5 - \sqrt{5}}{2} \right\} = \{3.62, 1.38\}$



d) $-0.2(x-3)^2 + 1 = \frac{1}{4}x + \frac{1}{2} \quad | -\frac{1}{4}x - \frac{1}{2}$
 $-\frac{1}{5}(x^2 - 6x + 9) + 1 = \frac{1}{4}x + \frac{1}{2} \quad | \cdot (-1)$

$-\frac{1}{5}x^2 + (\frac{6}{5} - \frac{1}{4})x - \frac{9}{5} + 1 - \frac{1}{2} = 0$

$-\frac{1}{5}x^2 + \frac{24-5}{20}x - \frac{6.5}{5} = 0 \quad | \cdot 20$

$-4x^2 + 19x - 26 = 0 \quad | \cdot (-1)$

$4x^2 - 19x + 26 = 0$

$a=4 \quad b=-19 \quad c=26$

$D = b^2 - 4ac = 19^2 - 4 \cdot 4 \cdot 26 < 0$
 \rightarrow keine Lösung!

