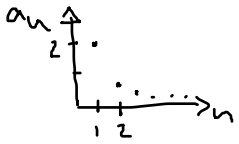
Aufgabe 1: Untersuchen Sie die Folgen auf Monotonie.

a) $a_n = \frac{2}{n^2}$



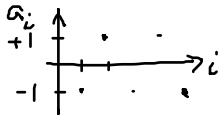
$\Delta a = a_{n+1} - a_n = \frac{2}{(n+1)^2} - \frac{2}{n^2} = ? \cdot \frac{n^2 - (n+1)^2}{(n+1)^2 n^2}$

$= ? \cdot \frac{n^2 - n^2 - 2n - 1}{(n+1)^2 n^2} = (-2) \cdot \frac{2n+1}{(n+1)^2 n^2}$ $\quad >0$ $\quad >0$

>0

$\Delta a < 0 \rightarrow$ streng monoton fallend

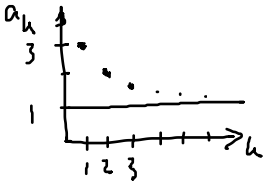
b) $a_i = (-1)^i$



keine Monotonie (unbest. divergent)

c) $a_k = \frac{k+2}{k}$

$\lim_{k\to\infty} \frac{k+2}{k} = 1$



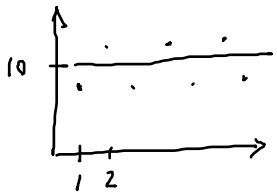
$\Delta a = a_{k+1} - a_k = \frac{k+3}{k+1} - \frac{k+2}{k}$

$= \frac{k(k+3) - (k+1)(k+2)}{(k+1)k} = \frac{k^2+3k - (k^2+3k+2)}{(k+1)k}$

$= \frac{k^2+3k-k^2-3k-2}{(k+1)k} = -\frac{2}{(k+1)k} < 0$

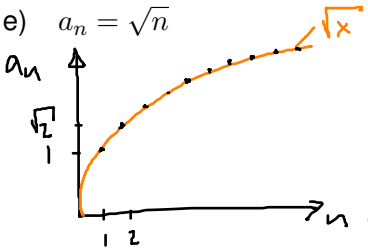
$\Delta a < 0 \rightarrow$ streng monoton fallend

d) $a_n = (-1)^n + 10$



$\rightarrow$ keine Monotonie

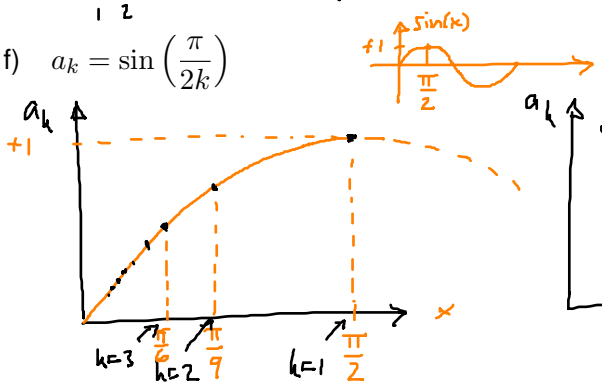
e) $a_n = \sqrt{n}$



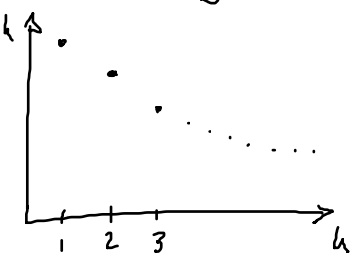
$\Delta a = \sqrt{n+1} - \sqrt{n} > 0$

streng monoton steigend

f) $a_k = \sin\left(\frac{\pi}{2k}\right)$

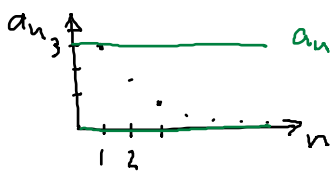


streng monoton fallend



Aufgabe 2: Untersuchen Sie die Folgen auf Beschränktheit.

a) $a_n = \frac{1}{2} \cdot a_{n-1}$ mit $a_1 = 3$

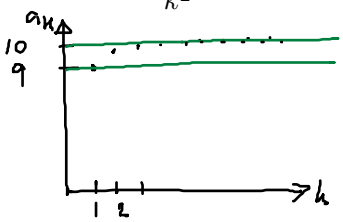


$a_n \leq 3$ (ob. Grenze)

$a_n > 0$ (unt. Grenze)

} beschränkt

b) $a_k = 10 - \frac{1}{k^2}$



$a_k < 10$ ob. Grenze

$a_k \geq 9$ unt. Grenze

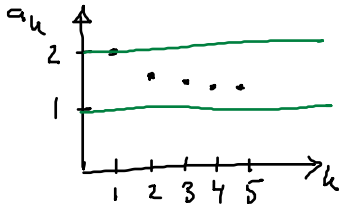
} beschränkt

c) $a_n = (-1)^{(n-1)} \cdot 2^{(n-1)}$ $(a_n) = 1, -2, +4, -8, +16, -32, \ldots$

"explodiert" zu $\pm\infty$ → keine Beschränkung

d) $(a_k) = 2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$ $k=2$

$a_k = \frac{k+1}{k}$ $\lim_{k\to\infty} \frac{k+1}{k} = 1$

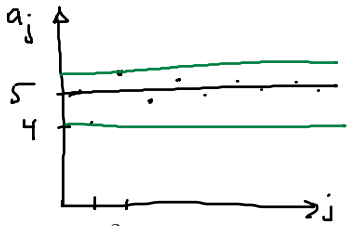


$a_k \leq 2$ ob. Grenze

$a_k > 1$ unt. Grenze

} beschränkt

e) $a_j = 5 + \frac{(-1)^j}{j}$ $\Rightarrow (a_j) = 4, 5.5, 4\frac{2}{3}, 5{,}25, \ldots$



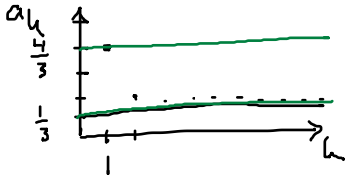
$a_j \leq 5.5$ ob. Grenze

$a_j \geq 4$ unt. Grenze

} beschränkt

f) $a_k = \frac{k^2+3}{3k^2}$

$a_k = \frac{1 + \frac{3}{k^2}}{3} = \frac{1}{3} + \frac{1}{k^2}$ $\to 0$



$a_k \leq \frac{4}{3}$ ob. Grenze

$a_k > \frac{1}{3}$ unt. Grenze

} beschränkt