

Gegeben sind die folgenden Glieder. Schreiben Sie die ersten sieben Glieder der Folge auf.

a) $a_3 = \frac{16}{4^2}$, $a_4 = \frac{25}{5^2}$, $a_5 = \frac{36}{6^2}$ $a_1 = 2^2 = 4$, $a_2 = 3^2 = 9$
 $(a_n) = 4, 9, 16, 25, 36, 49, 64$

b) $a_1 = +1$, $a_2 = -1$, $a_3 = +1$, $a_4 = -1$
 $(a_n) = +1, -1, +1, -1, +1, -1, +1$

c) $c_6 = \frac{7}{12}$, $c_7 = \frac{4}{7}$, $c_8 = \frac{9}{16}$ $c_6 = \frac{6+1}{2 \cdot 6} = \frac{7}{12}$ $c_1 = \frac{1+1}{2 \cdot 1} = \frac{2}{2}$ $c_2 = \frac{2+1}{2 \cdot 2} = \frac{3}{4}$
 $c_7 = \frac{8}{14}$ $(c_n) = \frac{2}{2}, \frac{3}{4}, \frac{4}{6}, \frac{5}{8}, \frac{6}{10}, \frac{7}{12}, \frac{8}{14}, \frac{9}{16}$

d) $b_1 = 4$, $b_2 = 7$, $b_3 = 12$, $b_4 = 19$, $b_5 = 28$, $b_6 = 39$
 $(b_n) = 4, 7, 12, 19, 28, 39, 52, \dots$

e) $p_3 = 12$, $p_4 = 6$, $p_5 = 3$, $p_6 = 1.5$
 $(p_n) = 48, 24, 12, 6, 3, 1.5, 0.75$

Schreiben Sie die ersten fünf Glieder für die folgenden expliziten Definitionen auf.

a) $a_n = 5n + 1$ $a_1 = 5 \cdot 1 + 1 = 5 + 1 = 6$ $a_2 = 5 \cdot 2 + 1 = 11$

$$(a_n) = 6, 11, 16, 21, 26, \dots$$

b) $a_n = \frac{1}{2^{4-n}}$ $a_1 = \frac{1}{2^{4-1}} = \frac{1}{2^3} = \frac{1}{8}$ $a_2 = \frac{1}{2^{4-2}} = \frac{1}{4}$ $a_3 = \frac{1}{2^{4-3}} = \frac{1}{2}$

$$(a_n) = \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \dots$$

c) $b_n = (-2)^{n-1}$ $b_1 = (-2)^{1-1} = (-2)^0 = 1$ $b_2 = (-2)^{2-1} = -2$ $b_3 = (-2)^2 = 4$

$$(b_n) = 1, -2, 4, -8, 16, -32, \dots$$

d) $e_k = \frac{k}{n+1}$ $e_1 = \frac{1}{n+1}$ $e_2 = \frac{2}{n+1}$

$$(e_k) = \frac{1}{n+1}, \frac{2}{n+1}, \frac{3}{n+1}, \frac{4}{n+1}, \frac{5}{n+1}, \dots$$

e) $f_j = \frac{j-1}{j^2+1}$ $f_1 = \frac{1-1}{1^2+1} = \frac{0}{2} = 0$ $f_2 = \frac{2-1}{4+1} = \frac{1}{5}$ $f_3 = \frac{3-1}{9+1} = \frac{2}{10}$

$$(f_j) = 0, \frac{1}{5}, \frac{2}{10}, \frac{3}{17}, \frac{4}{26}, \frac{5}{37}, \dots$$

Bestimmen Sie die explizite Definition der folgenden Folgen.

a) $3, 7, 11, 15, \dots$ $a_1 = 4 \cdot 1 - 1 = 3$ $a_3 = 4 \cdot 3 - 1 = 11$
 $a_2 = 4 \cdot 2 - 1 = 7$
 $a_n = 4n - 1$

b) $-3, +6, -9, +12, -15, \dots$ $a_1 = (-1)^1 \cdot 3 \cdot 1 = -3 \checkmark$ $a_2 = (-1)^2 \cdot 3 \cdot 2 = +6 \checkmark$ $a_3 = (-1)^3 \cdot 3 \cdot 3 = -9 \checkmark$
 $a_n = (-1)^n \cdot 3n$

c) $\frac{3}{7}, \frac{4}{6}, 1, \frac{6}{4}, \frac{7}{3}, \dots$ $a_1 = \frac{1+2}{8-1} = \frac{3}{7}$ $a_3 = \frac{3+2}{8-3} = \frac{5}{5} = 1$
 $a_2 = \frac{2+2}{8-2} = \frac{4}{6}$ $a_5 = \frac{5+2}{8-5} = \frac{7}{3}$ $a_n = \frac{n+2}{8-n}$

d) $1, 2, 6, 24, 120, \dots$ $a_1 = 1! = 1$ $a_2 = 2! = 1 \cdot 2 = 2$ $a_3 = 3! = 1 \cdot 2 \cdot 3 = 6$ $a_4 = 4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$
 $a_n = n!$

e) $0, 3, 2, 5, 4, 7, 6, \dots$ $a_n = n + (-1)^n$ $a_1 = 1 + (-1)^1 = 1 - 1 = 0 \checkmark$ $a_2 = 2 + (-1)^2 = 2 + 1 = 3 \checkmark$ $a_3 = 3 + (-1)^3 = 3 - 1 = 2 \checkmark$

f) $a_3 = 1.1, a_4 = \frac{8}{7}, a_5 = \frac{5}{4}$ $a_3 = \frac{20-3 \cdot 3}{19-3 \cdot 3}$ $a_5 = \frac{20-3 \cdot 5}{19-3 \cdot 5} = \frac{5}{4}$
 $a_4 = \frac{20-3 \cdot 4}{19-3 \cdot 4} = \frac{8}{7}$ $a_n = \frac{20-3 \cdot n}{19-3 \cdot n}$

g) $d_3 = \frac{5}{4}, d_4 = \frac{7}{8}, d_5 = \frac{9}{16}$ $d_n = \frac{2n-1}{2^{n-1}}$ $d_1 = \frac{2 \cdot 1 - 1}{2^{1-1}} = \frac{1}{1} = 1$ $d_5 = \frac{2 \cdot 5 - 1}{2^{5-1}} = \frac{10-1}{2^4} = \frac{9}{16}$
 $d_3 = \frac{2 \cdot 3 - 1}{2^{3-1}} = \frac{5}{4}$

h) $h_1 = 0, h_2 = \frac{1}{6k}, h_5 = \frac{16}{15k}, h_7 = \frac{36}{21k}$ $h_n = \frac{(n-1)^2}{3nk}$ $h_7 = \frac{(7-1)^2}{3 \cdot 7 \cdot k} = \frac{36}{21k}$